Lesson 012
Expected Values

Friday, October 6

If you bet $100 on a coin toss,
what do you expect to happen?

If you flip a coin 100 times, how many heads do you expect to observe?

Expected Value

- The **expected value** is the average of a **probability distribution**.
- Also referred to as the **expectation**.
- For a random variable, X, this is denoted $E[X]$.

Connection to Sample Mean

$$\{1,2,2,3,7\}$$

$$\bar{x} = \frac{1 + 2 + 2 + 3 + 7}{5} = \frac{15}{5} = 3$$

Connection to Sample Mean

$$\bar{x} = \frac{1+2+2+3+7}{5} = \frac{15}{5} = 3$$

$$\bar{x} = \frac{1}{5}(1) + \frac{2}{5}(2) + \frac{1}{5}(3) + \frac{1}{5}(7)$$

Connection to Sample Mean

$$\{1,2,2,3,7\} \quad p(x) = \begin{cases} \frac{1}{5} & x = 1 \\ \frac{2}{5} & x = 2 \\ \frac{1}{5} & x = 3 \\ \frac{1}{5} & x = 7 \\ 0 & \text{otherwise} \end{cases}$$

Connection to Sample Mean

$$X \sim p(x) \implies E[X] = 3$$

Connection to Sample Mean

$$E[X] = (1)P(X = 1) + (2)P(X = 2) + (3)P(X = 3) + (7)P(X = 7)$$

$$= (1)p(1) + (2)p(2) + (3)p(3) + (7)p(7)$$

Connection to Sample Mean

$$E[X] = \frac{1}{5}(1) + \frac{2}{5}(2) + \frac{1}{5}(3) + \frac{1}{5}(7)$$

$$= 3$$

Expected Value

- For $X \sim p(x)$ we get

$$E[X] = \sum_i x_i p(x_i)$$

- In general $E[X] \neq \bar{x}$.

A particular illness is treated with a course of treatments, until symptoms subside. Cases require 1, 2, 3, or 4 rounds of treatment in 20%, 30%, 40%, and 10% of situations, respectively. How many treatments are expected for a newly diagnosed individual?

- 3 — 0%
- 2.4 — 0%
- 2.5 — 0%
- 2 — 0%

What is the expected value of the Bernoulli distribution? $p(x) \begin{cases} p & x = 1 \\ 1 - p & x = 0 \\ 0 & \text{otherwise} \end{cases}$

p — 0%

$1 - p$ — 0%

1 — 0%

$\frac{p}{1-p}$ — 0%

Properties of Expectations

- For $X \sim p(x)$ and a function h, we get
$$E[h(X)] = \sum_i h(x_i)p(x_i)$$
- In general $E[h(X)] \neq h(E[X])$.

Expectations of Linear Functions

- If $h(X) = a + bX$, then

$$E[h(X)] = a + bE[X]$$

- Example: T is temperature in Celsius, $F = 1.8T + 32$. Then

$$E[F] = 32 + 1.8E[T]$$

The expected weight of a part produced in a manufacturing plant is $E[X] = 4.5$kg. The manufacturer begins dealing with a customer who wants the weight in pounds (1kg$=2.2$lbs). What is the expected weight in pounds?

9.9

0%

2.05

0%

6.7

0%

Not enough information is given.

0%

A die manufacturer is considering the variability in their process. They find that the expected length of a side on dice that they produce is 1.6cm. What is the expected volume of the die, assuming that the process always produces exactly cubic dice?

$1.6^3 = 4.096$

0%

$3 \times 1.6 = 4.8$

0%

$1.6^2 = 2.56$

0%

Not enough information is provided.

0%

Variance

- Recall that the **sample variance** was the **sample average** of squared deviations from the mean.
- The **population variance** or **variance of a distribution** is the analogous quantity, with expectations.

Variance

- For $X \sim p(x)$ we get

$$\text{var}(X) = E\left[(X - E[X])^2\right]$$

- This is $E[h(X)]$ for $h(X) = (X - E[X])^2$.

$$\text{var}(X) = E[X^2] - E[X]^2$$

A particular illness is treated with a course of treatments, until symptoms subside. Cases require 1, 2, 3, or 4 rounds of treatment in 20%, 30%, 40%, and 10% of situations, respectively. What is the variance in the number of treatments required?

$6.6 - 2.5^2 = 0.35$

0%

2.4

0%

$6.6 - 2.4^2 = 0.84.$

0%

6.6

0%

What is the variance of the Bernoulli distribution? $p(x) \begin{cases} p & x = 1 \\ 1-p & x = 0 \\ 0 & \text{otherwise} \end{cases}$

p — 0%

$p(1-p)$ — 0%

1 — 0%

p^2 — 0%

Properties of Variance

- For $X \sim p(x)$ and a function h, we get

$$\text{var}\{h(X)\} = E\left[(h(X) - E[h(X)])^2\right]$$

- In general $\text{var}\{h(X)\} \neq h(\text{var}(X))$.

Variance of Linear Functions

- If $h(X) = a + bX$, then

$$\text{var}\{h(X)\} = b^2\text{var}(X)$$

- Example: T is temperature in Celsius, $F = 1.8T + 32$. Then

$$\text{var}(F) = 3.24 \cdot \text{var}(T)$$

A company has a weekly pay variance of 100. Different raise structures are considered. In one, they would increase everyone's pay by $\$5$ per week, in the other they would give a 5% raise to each individual. What happens to the variance?

After the raises, each setting will produce the same pay variance, and this will increase from the current level. — 0%

After the raises, the first option (constant raise) will produce a higher pay variance than the second. — 0%

After the raises, the second option (percentage raise) will produce a higher pay variance than the first. — 0%

After the raises, each setting will produce the same pay variance, and this will decrease from the current level. — 0%

Known Distributions

- If $X \sim \text{Bern}(p)$ then

$$E[X] = p \text{ and } \text{var}(X) = p(1-p)$$

- If $X \sim \text{Geo}(p)$ then

$$E[X] = \frac{1}{p} \text{ and } \text{var}(X) = \frac{1-p}{p^2}$$